

## INVESTIGATION OF A FLOW AROUND CAVITIES

A. I. Shvets

UDC 533.06.11.72

*A supersonic flow around a ring cavity on a cylindrical body has been experimentally investigated in wide ranges of relative lengths of the cavity and Mach numbers. The influence of these parameters on the pressure distribution, the structure of the flow, and the regimes of flow around the cavity has been determined.*

A flow in cavities (hollows, depressions) can arise in various cases, for example, when they are positioned on the elements of aircrafts, industrial plants, or wind tunnels, around which flow flows. There are a large number of theoretical and experimental works in which a flow around cavities has been investigated in a wide range of incoming flow velocities (from the velocity of an incompressible fluid flow to the hypersonic Mach numbers). These works were carried out in two main directions: (1) averaged flows in cavities and (2) pressure pulsations and acoustic processes.

In [1], the characteristics of a fluid flow in plane rectangular cavities have been investigated in the range of Reynolds numbers 500–5000. Flows of an incompressible fluid in shallow and deep cavities of rectangular cross section have been investigated in [2] for the laminar regime of flow upstream of the cavity. The experiments conducted in [3] have shown that turbulization of the external flow significantly increases the rate of recirculation motion. The limited data on a flow around cavities have been obtained at subsonic and small supersonic flow velocities; for example, the results of numerical calculations of a compressible viscous gas flow around cavities in the range of  $M = 1.05$ – $1.2$  are presented in [4].

A number of investigations have been carried out in a supersonic range of flow velocities. The numerical calculations of unsteady flows in rectangular cavities at supersonic velocities of the external flow are presented in [5]. In [6] it is proposed to use passive methods of control of flow for obtaining a beneficial distribution of the pressure in a closed cavity. A flow around models of cavities at  $M = 2.78$  [7] and a supersonic flow around a step and a cavity have been investigated in [8]. In [9–13], the results of experimental investigations of the pressure distribution, the structure of the flow, and the heat exchange in the cavities are presented. A flow around a cavity on an axisymmetric body has been investigated in [14] in the range of hypersonic Mach numbers, particularly at  $M = 7.3$  [14].

It should be noted that in the above-mentioned investigations of flows around cavities, only fragmentary data on some geometric parameters have been obtained and most attention has been concentrated on the study of cavities with small lengths in which a regime of flow with an open separation zone is realized. In contrast to these works, we have investigated, using an automated model, a flow around cavities in a wide range of Mach numbers and in a range of cavity lengths, in which both regimes of flow are realized.

The experiments were performed in a supersonic wind tunnel with a perforated working section, providing a means for conducting tests in wide ranges of Mach ( $M = 0.4$ – $4.0$ ) and Reynolds ( $Re = 4 \cdot 10^5$ – $2 \cdot 10^7$ ) numbers. In the experiments, we used a model in the form of a cylindrical body 68 mm in diameter with a conical head ( $\theta = 9^\circ$ ). A cavity of depth  $D = 10$  mm with a pressure transducer positioned in it was located at a distance of 280 mm from the conical part. The central cylindrical part of the model was moved automatically, which made it possible to change the length of the cavity  $L$  from 10 to 140 mm with a step of 10 mm. Moreover, the inner cylindrical part of the model was moved automatically, which made it possible to move the pressure transducer and measure the pressure distribution over the bottom wall of the cavity in the range of  $X/L$  from 0 to 1 during one experiment. The relative mean-square error in measuring the pressure was 0.03. The flow was photographed by a Toepler device.

The total pressure in the vicinity of the model at a distance of 50 mm from the cylindrical surface near the leading edge of the cavity and the static pressure on the surface of the model were measured using a Pitot tube (of

---

M. V. Lomonosov Moscow State University, Moscow, Russia. Translated from *Inzhenerno-Fizicheskii Zhurnal*, Vol. 76, No. 6, pp. 13–18, November–December, 2003. Original article submitted October 3, 2002; revision submitted February 17, 2003

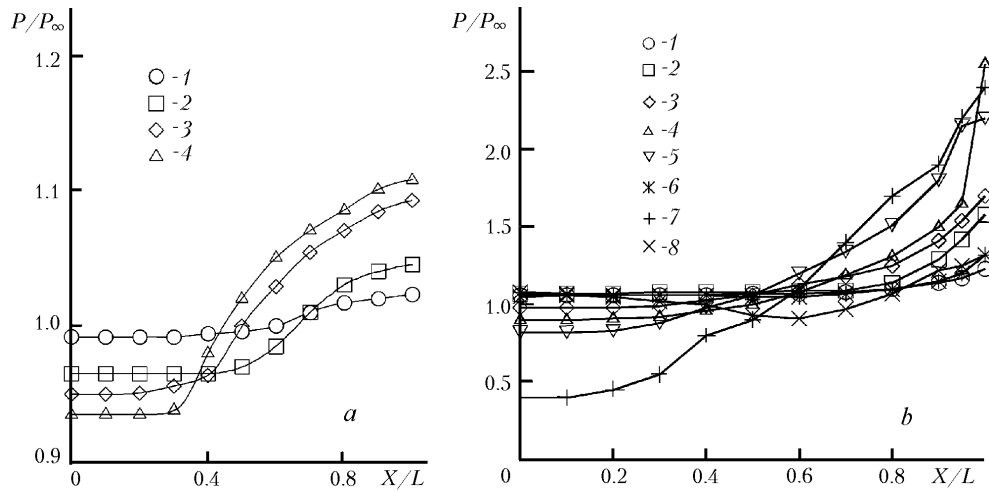


Fig. 1. Distribution of the relative pressure  $P/P_\infty$  over the bottom of the cavity: a)  $M_e = 0.6$ :  $L/D = 5.3$  (1),  $7.3$  (2),  $9.4$  (3), and  $11.3$  (4); b)  $M_e = 1.9$ :  $L/D = 5.3$  (1),  $7.3$  (2),  $9.4$  (3),  $10.4$  (4),  $12.5$  (5); 6) calculation,  $L/D = 6$ ,  $M = 1.5$ ,  $Re = 6.5 \cdot 10^6$  [12]; 7) experiment,  $L/D = 6$ ,  $M = 1.5$  [10]; and 8) experiment,  $L/D = 9$ ,  $M = 1.5$  [11].

diameter 0.8 mm). From this data, using the Rayleigh formula, we determined the number  $M_e$  upstream of the leading edge of the cavity. The Reynolds number  $Re_L$  in the range of  $M_e = 0.6$ – $2.84$  was changed from  $Re_L = 0.6 \cdot 10^7$  at  $M_e = 0.6$  to  $Re_L = 1 \cdot 10^6$  at  $M_e = 2.84$ . The measurements of the boundary-layer profile have shown that the boundary layer upstream of the cavity is turbulent throughout the range of Mach numbers under study. The thickness of the boundary layer  $\delta$  is equal to 7–8 mm at  $M_e = 0.6$  and 0.8, and  $\delta = 6$  mm at  $M_e = 2.84$ .

In the experiments, we performed photographing and filming of the flow with a large and small time of exposure. At a time of exposure  $t = 10$  msec, which was much larger than the cycle of pulsations, a time-averaged pattern of the flow was recorded. At  $t = 1$   $\mu$ sec, the oscillations of flow and motion of vortex formations were recorded.

**Pressure Distribution over the Bottom of the Cavity.** In the case of a supersonic gas flow around cavities, two regimes of flow are realized. The critical value of  $L/D$ , at which an open cavity transforms into a closed one, has been determined in [6]. If the ratio of the length of the cavity to its depth is smaller than a certain value, the cavity is open — the separation zone occupies the entire cavity. The other regime of flow — closed cavity — is realized when the ratio  $L/D$  exceeds any critical value and the second separation zone arises near the back wall. In the case of a supersonic flow and a turbulent boundary layer, the boundary value at which the regime of flow in an open cavity changes to the regime of flow in a closed cavity lies within  $L/D = 10$ – $13$ .

The experimental investigations of a flow around a cavity were carried out in a wide range of its relative lengths. For a description of the pressure distribution over the bottom part of the cavity, we have selected two characteristic cases of flow around it: one for subsonic Mach numbers and another for supersonic ones.

*Subsonic flow.* Figure 1a shows the pressure distribution  $P/P_\infty$  over the bottom of a ring cavity at  $M_e = 0.6$  versus the relative distance to the front point  $X/L$ . The curves can be divided into two portions corresponding to the case where  $P/P_\infty = \text{const}$  downstream of the front wall of the cavity and the case where the pressure is increased upstream of the back wall. At  $M_e = 0.6$ , the pressure at the bottom of comparatively short cavities ( $L/D = 5.3$ ) remains constant up to the distance  $X/L = 0.6$ . As  $L/D$  increases, the region of constant pressure decreases and, at  $L/D = 11.3$ , the ratio  $X/L = 0.2$ . For all the values of  $L/D$ , the pressure increases as the back wall is approached, and the pressure  $P/P_\infty$  near the wall increases from 1.02 to 1.11 as  $L/D$  increases.

In the case of a subsonic flow around open cavities, the pressure distribution is determined by the three main mechanisms: vortices in the cavity, interaction of the boundary layer with the external flow, and stagnation of the flow near the back wall. If  $L/D$  is small, the vortices and viscous interaction play a dominant role. As  $L/D$  increases, the compression of the flow near the back wall and diffusion of kinetic momentum through the boundary layer become predominant. This leads to an increase in the pressure at the bottom of the cavity, beginning with  $X/L = 0.5$ , i.e., before the compression process begins near the back wall.

In the experiments, we have obtained photographs of the flow around cavities at subsonic Mach numbers. In the case of an open cavity, the boundary layer transforms into a free viscous layer and, after separation from the leading edge, first moves away from the cavity and then approaches it. The layer upstream of the trailing edge expands. One large vortex occupies practically the entire cavity. The pictorial photographs allow us to suggest that near the lower corner of the front wall there arises another small vortex rotating in the opposite direction. In the case of a closed cavity, the boundary layer separates from the leading edge, approaches the bottom of the cavity, and then moves away from it and passes over the trailing edge. This regime is characterized by the existence of separate, isolated zones of circulation flow near the front and back walls of the cavity; in this regime of flow, the extent of the leading vortex is much larger than the extent of the trailing vortex.

The structure of a subsonic flow around an open cavity with a large value of  $L/D$  differs from that of a supersonic flow because, in the latter case, compression shocks do not arise in the cavity and the disturbances move upstream from the back separation zone. At the same time, flows with one or two separation zones arise and there is a range of  $L/D$  values in which the open-cavity regime changes to a closed-cavity regime. The  $L/D$  values at which the regime changes, in addition to the length of the cavity, are determined by the parameters of the boundary layer at the leading edge of the cavity.

*Supersonic flow.* Figure 1b shows the experimental data for a supersonic flow around a cavity at  $M_e = 1.9$ . For  $L/D = 5.3$ , the pressure remains constant at a distance from the front point of the cavity bottom to  $X/L = 0.7$ , and then it increases and reaches  $P/P_\infty = 1.22$  near the back wall. In an open cavity, the region of minimum pressure is positioned near the front wall of the cavity because of the propagation of rarefaction waves from the leading edge. A high pressure propagates up from the compression region near the back wall along the subsonic part of the boundary layer, with the result that the external flow is forced out and the pressure in the cavity increases. As the length of the cavity increases, the pressure in its front part and the region of constant pressure decrease. At the same time, with increase in  $L/D$  the pressure near the back wall increases from  $P/P_\infty = 1.6$  for  $L/D = 7.3$  to  $P/P_\infty = 2.57$  for  $L/D = 12.5$ . It should be noted that in the case where  $M_e = 2.84$ , in contrast to the case where  $M_e = 1.9$ , approximately to the cavity length  $L/D = 10.0$ , the pressure in the central part of the cavity decreases, beginning with the front point, and then increases when the back wall is approached.

The length at which an open cavity transforms into a closed one can be determined from a comparison of the data on the pressure distribution and the data of optical measurements. The cavities with  $L/D = 0.5$ –14 were tested at  $M_e = 2.84$ , and the cavities with  $L/D = 4.3$ –12.5 were tested at  $M_e = 1.9$ . If in the first case the change from the open separation zone to a closed one took place in the range of  $L/D$  from 12 to 14, in the second case only the regime of flow with an open separation zone was realized in the range of cavity lengths under study. In the closed regime of flow, the increased pressure at the trailing edge of the cavity is not transferred to its leading edge. The flow downstream of the leading edge expands, rotates, moves to the bottom, and, when the bottom is approached, turns around along it. In this region there arise compression waves and the pressure increases. The pressure increases most abruptly near the back wall in the range of  $X/L = 0.95$ –1.00, where it reaches the value of  $P/P_\infty = 3.3$ .

In the case where  $M_e = 1.9$ , in all the cavities studies, including the cavity with a ratio  $L/D$  close to the critical value (10–13), the pressure distribution changes monotonically because of the interaction of the separation zones. At the same time, it has been established in the experiments performed at  $M_e = 2.84$  that the change of an open separation zone to a closed one ( $L/D = 14$ ) leads to an abrupt change in the shape of the pressure curves at the bottom, on which there appear two clearly defined deflections corresponding to the first compression shock arising when the flow turns parallel to the bottom wall downstream of the rarefaction wave at the leading edge of the cavity and the second compression shock arising upstream of the back separation zone.

Figure 1b shows the results of calculations done in [12] at  $L/D = 6$ ,  $M = 1.5$ , and  $Re = 6.5 \cdot 10^6$ . In this work, a supersonic turbulent flow around rectangular cavities with a relative length of 6 and 17.5 and a width of 1 and 2.5, respectively, has been investigated. The two-dimensional Navier–Stokes equations were solved by the McCormack scheme. This figure also shows the results of the experiments conducted in [10] at  $L/D = 6$ ,  $M = 1.5$ , and  $Re = 6.5 \cdot 10^6$ . In [10], a supersonic turbulent flow around a two-dimensional rectangular cavity with a relative length  $L/D = 6$ , 12, and 16 has been investigated under the conditions of a thick boundary layer. The flow in the cavity was modeled using a two-dimensional Navier–Stokes equation. Figure 1b also shows the results of experiments conducted in [11] at  $L/D = 9$  and  $M = 1.5$ , in which the flow in a two-dimensional rectangular cavity has been investigated theo-

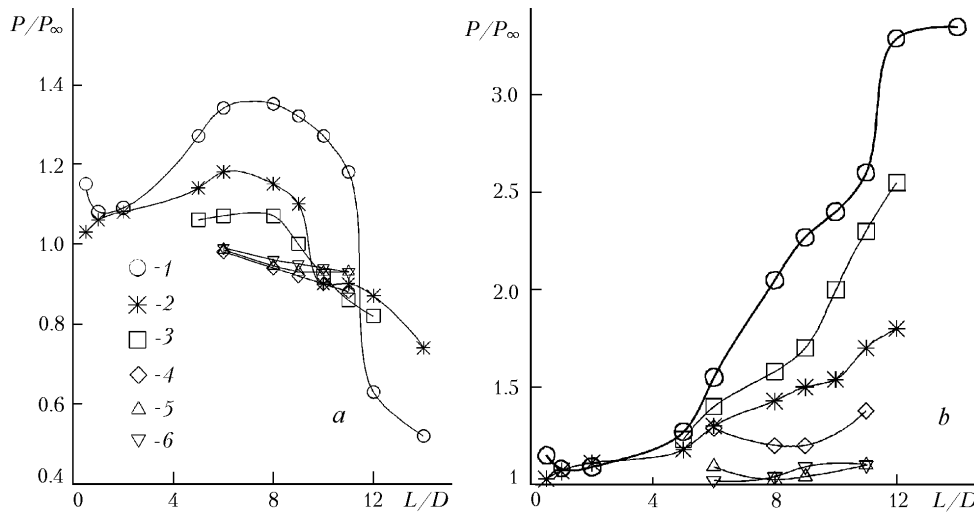


Fig. 2. Change in the relative pressure  $P/P_\infty$  at the bottom of the cavity depending on the relative length of the cavity  $L/D$ : a) front point  $X/L = 0$ ; b) back point  $X/L = 1$  ( $M_e = 2.84$  (1) and  $2.78$  (2), experiment in [7];  $1.9$  (3);  $1.2$  (4);  $0.8$  (5); and  $0.6$  (6)).

retically and experimentally under the conditions of a thick boundary layer at Mach numbers 1.5 and 2.5. For open cavities, the numerical solutions of the complete Navier–Stokes equations for the pressure distribution over the bottom part of the cavity [1] are in qualitative agreement with the experimental results obtained in [10].

It should be noted that the calculated values of the pressure, obtained in [10], are at variance with the experimental data. The main reason for this discrepancy is the three-dimensional character of the flow investigated in the experiments ( $W/D = 5.5$ ). The influence of a three-dimensional flow on the pressure distribution in the cavity was also detected in the transient regime of flow ( $10 < L/D < 13$ ) around cavities of different width [12]. It has been established that this influence is more significant for a closed cavity. This partially explains the fact that the calculation and experimental data agree only qualitatively for closed cavities and there is a quantitative agreement between them for open cavities.

**Influence of the Relative Length of the Cavity.** Let us consider the dependence of the structure of the flow in a cavity on its relative length. It is interesting to analyze the change in the pressure  $P/P_\infty$  at two characteristic points depending on the relative length  $L/D$  of the cavity:  $X/L = 0$ , downstream of the leading edge, and  $X/L = 1$ , upstream of the trailing edge. The pressures at these points characterize the change in the structure of the flow in the cavity at different relative lengths  $L/D$  and different Mach numbers.

Figure 2a shows the data of experiments for the front point of the bottom part of the cavity ( $X/L = 0$ ). At a Mach number  $M_e = 2.84$  (the largest for these experiments) the pressure in a narrow cavity somewhat decreases when  $L/D$  increases from 0.5 to 1.0. Open cavities can be divided into shallow and deep cavities [8]. The boundary at which the regime of flow in a shallow cavity changes to the regime of flow in a deep cavity corresponds to  $L/D = 1$ . The deep cavities are characterized by the existence of an oscillating free viscous layer and can generate resonance oscillations. For  $M_e = 2.84$ , an increase in the relative length  $L/D$  from 1 to 6 leads to an increase in the pressure. In the case of a supersonic flow around an open cavity, the incoming flow separates in the vicinity of the leading edge of the cavity and attaches to the trailing edge. A part of the gas is involved in the circulation flow and turns back to the front wall of the cavity. The free viscous layer is forced out to the external flow as the trailing edge is approached.

An increase in the relative length of the cavity  $L/D$  from 8 to 14 leads to a decrease in the pressure.  $P/P_\infty$  decreases sharply when  $L/D$  increases from 11 to 12, which is due to the change from an open separation zone to a closed one. Figure 2a also shows the data of experiments [7] on a flow around cavities at  $X/L = 0$  and  $M = 2.78$  under the conditions of a turbulent boundary layer. It is shown in [7] that at small  $Re$  numbers the length of the first separation zone for an axisymmetric flow is larger than that for a plane flow.

In the regime of flow with a closed separation zone, the flow downstream of the front wall is similar to the bottom flow. For the model used, this flow is the bottom flow downstream of the back cylindrical step. The results of

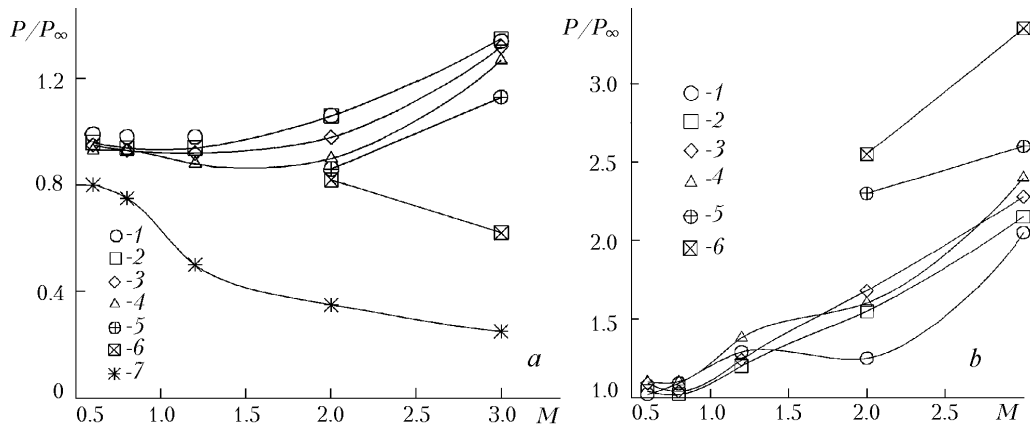


Fig. 3. Dependence of the relative pressure  $P/P_\infty$  at the bottom of the cavity on the Mach number: a) front point  $X/L = 0$ ; b) back point  $X/L = 1$ ;  $L/D = 5.3$  (1), 7.3 (2), 9.4 (3), 10.4 (4), 11.3 (5), and 12.5 (6); and 7) experiment, bottom pressure downstream of the cone [15].

investigations of this flow allow the conclusion that in the range of moderate values of  $M = 2-3.5$ , at a large Reynolds number the ratio of the length of the first separation zone to the height  $D$  is practically independent of  $M$  and  $Re$  for both the laminar and turbulent boundary layers.

At  $M_e = 1.9$  and  $M_e = 2.84$ , the dependence  $P/P_\infty = f(L/D)$  changes in a similar way: the pressure increases and then decreases when  $L/D$  increases from 4.3 to 8. For the small supersonic Mach number  $M_e = 1.2$ , an increase in the relative length of the cavity  $L/D$  from 4.3 to 9.8 leads to a decrease in the pressure. In the case of subsonic Mach numbers ( $M = 0.8$  and  $0.6$ ),  $P/P_\infty$  also decreases with increase in  $L/D$  from 5.3 to 11.3.

At the back point of the cavity ( $X/L = 1$ ), the influence of the relative length  $L/D$  on  $P/P_\infty$  is shown in Fig. 2b. As for the front point, an increase in  $L/D$  from 0.5 to 1.0 leads to a decrease in the pressure, and a further increase in the length of the cavity leads to an increase in it. Contrary to the front point, at which the pressure decreases after  $L/D = 6$ , at the back point the pressure increases throughout the range of  $L/D$ .

At  $M_e = 1.9$ , the pressure at the back point increases in the range of  $L/D = 5.3-12.5$ . It is interesting to note that, at the small supersonic flow velocity  $M_e = 1.2$ , a pressure minimum is realized at  $L/D = 8-9$ . At the subsonic flow velocity  $M = 0.6$ , the pressure increases only slightly.

In the case of a closed cavity, the characteristics of the second separation zone upstream of the back wall are similar to the characteristics of the compression zone upstream of the step. The maximum pressure  $(P/P_\infty)_{\max}$  at the beginning of the compression zone depends on the free interaction arising as a result of separation of the boundary layer. As the back wall is approached, the pressure increases, which causes a deflection of the external flow. The data for  $P_{\max}/P_\infty$  at the point  $X/L = 1$ , obtained in [7] for the flow around a cavity at  $M = 2.78$ , are also shown in Fig. 2a.

**Influence of the Mach Number.** We now consider the dependences of the pressure at the characteristic points of the cavity on the Mach number of the incoming flow. The relative pressures at the front point ( $X/L = 0$ ) of the cavities of certain length are shown in Fig. 3a. For the cavities with a relative length  $L/D = 5.3-10.4$ , the pressure first decreases with increase in  $M$ , reaches a minimum at small supersonic flow velocities, and then increases with increase in  $M_e$  to 2.84. A different dependence is observed only for the longest cavity ( $L/D = 12.5$ ), in which the pressure decreases with increase in  $M_e$  from 1.9 to 2.84. For a closed cylindrical cavity, the pressure downstream of the front wall is comparable to the bottom pressure downstream of an axisymmetric body,  $L/D = \infty$  in the last case. For comparison, Fig. 3a shows the data [15] of investigation of the bottom pressure downstream of a cone with  $\theta = 7.5^\circ$ . It is seen from these data that throughout the range of Mach numbers under study the bottom pressure is much lower than the pressure at the point  $X/L = 0$  in the cavity with  $L/D = 12.5$ . It should also be noted that the axisymmetric model of a cavity was used in these experiments, whereas it has been shown in [12] that at small  $Re$  numbers the length of the first separation zone of an axisymmetric flow is larger than that of a plane flow.

The relative pressures  $P/P_\infty$  at the back point ( $X/L = 1$ ) of the cavities are shown in Fig. 3b. For all the cavity lengths, the pressure increases with increase in the Mach number throughout the range of Mach numbers under

study ( $M_e = 0.6\text{--}2.84$ ), except for the short cavity ( $L/D = 5.3$ ), in which a local decrease in the pressure arises when  $M_e$  is approximately equal to 1.9. The results of investigations of the axisymmetric and plane cavities [12] allow the conclusion that the main tendencies in their behavior are the same: an axisymmetric cavity closes at the same length as a plane cavity.

Let us consider the structure of a supersonic flow near the cavities in two regimes of flow — with an open and a closed separation zone. In an open cavity, the boundary layer separates at a corner point, in an external nonviscous flow there arises an expanding flow, and the free viscous layer moves to the bottom of the cavity. When the layer comes close to the bottom, it is deflected outward and a compression shock arises. A part of the free viscous layer turns to the front wall when the trailing edge is approached and a circulation flow arises in the cavity, and the other part of the layer is forced out to the external flow. At the trailing edge, the flow is deflected once again in the rarefaction wave parallel to the cylindrical surface of the model. In a closed cavity, the boundary layer moving to the bottom cut separates at a corner point and in doing so forms the first circulation zone. The flow moves to the bottom cut and turns parallel to the bottom wall. The change in the direction of the flow leads to the formation of the first compression shock. In this case, the back wall represents a ring bulge, the flow around which experiences rotation, with the result that a second compression shock is formed. Upstream of the back wall, a part of the gas enters the second circulation flow near the back wall. The free viscous layer attaches once again to the wall at a corner point, and in the external flow there arises a flow expansion in the rarefaction wave.

## CONCLUSIONS

1. Experimental investigations of a flow around a cavity have been carried out at Mach numbers from subsonic to supersonic in a wide range of cavity lengths.
2. The influence of the Mach number on the structure of the flow in a cavity has been investigated.
3. The influence of the length of the cavity on the flow around it, including the poorly understood regime of flow around a closed cavity, has been investigated.

## NOTATION

$L$  and  $D$ , length and depth of the cavity, mm;  $X$ , distance from the front wall of the cavity to the measurement point, mm;  $W$ , width of the cavity;  $X/D$ , relative length of the cavity;  $X/L$ , relative distance from the front wall of the cavity to the measurement point;  $X = 0$ , front point of the bottom part of the cavity;  $X = 1$ , back point of the bottom part of the cavity;  $\theta$ , angle of semi-opening of the cone;  $\delta$ , thickness of the boundary layer;  $M$ , Mach number;  $M_e$ , Mach number at the outer boundary of the boundary layer upstream of the leading edge of the cavity;  $Re$ , Reynolds number, m;  $Re_L$ , Reynolds number at the outer boundary of the boundary layer upstream of the leading edge of the cavity which is determined by the parameters of the incoming flow and the distance from the front point of the model to the leading edge of the cavity, m;  $P$ , measured pressure, Pa;  $P_\infty$ , static pressure of the undisturbed flow, Pa;  $P/P_\infty$ , relative pressure. Subscripts:  $\infty$ , conditions in an incoming undisturbed flow;  $e$ , conditions at the outer boundary of the boundary layer upstream of the leading edge of the cavity;  $\max$ , maximum value.

## REFERENCES

1. V. D. Zhak, V. A. Mukhin, and V. E. Nakoryakov, *Prikl. Mekh. Tekh. Fiz.*, No. 2, 54–59 (1981).
2. V. Ya. Bogatyrev and V. A. Mukhin, *Prikl. Mekh. Tekh. Fiz.*, No. 3, 70–74 (1984).
3. A. A. Bormusov, G. A. Glebov, A. N. Shchelkov, and R. A. Yakushev, *Izv. Akad. Nauk SSSR, Mekh. Zhidk. Gaza*, No. 2, 162–67 (1986).
4. L. G. Gvozdeva and Yu. P. Lagutov, *Izv. Akad. Nauk SSSR, Mekh. Zhidk. Gaza*, No. 3, 185–190 (1988).
5. N. L. Zaugol'nikov, M. A. Koval', and A. I. Shvets, *Izv. Akad. Nauk SSSR, Mekh. Zhidk. Gaza*, No. 2, 121–127 (1990).
6. F. J. Wilcox, Experimental investigation of porous-floor effects on cavity flow fields at supersonic speeds, NASA Langley Research Center. NASA-TP-3032. Hampton, VA (1990).

7. A. F. Charwat, J. N. Roos, F. C. Dewey, and J. A. Hitz, *J. Aero-Space Sci.*, **28**, No. 6, 457–470 (1961).
8. I. Tani, M. Iuchi, and H. Komoda, Experimental investigation of flow separation associated with a step or a groove, Aero. Research Institute. Tokyo Univ. Rep. No. 364 (1961).
9. W. L. Hankev and J. S. Shang, *AIAA J.*, **18**, No. 8, 392–396 (1980).
10. O. Baysal and S. Srinivasan, Unsteady Viscous Calculations of Supersonic Flows past Deep and Shallow Three-Dimensional Cavities, AIAA Paper. No. 88-0101, New York (1988).
11. X. Zhang and J. A. Edwards, *Aeronaut. J.*, **94**, No. 940, 355–364 (1990).
12. I. Kim and N. Chokani, *J. Aircraft*, **29**, No. 2, 217–223 (1992).
13. A. N. Batov, K. I. Medvedev, and Yu. A. Panov, *Izv. Ross. Akad. Nauk, Mekh. Zhidk. Gaza*, No. 4, 74–79 (1998).
14. A. Morgenstern and N. Chokani, *AIAA J.*, **32**, No. 12, 2387–2393 (1994).
15. A. I. Shvets, *Prog. Aerospace Sci.*, **18**, 177–208 (1978).